Separation Delay in Turbulent Boundary Layers via Model Predictive Control of Large-Scale Motions

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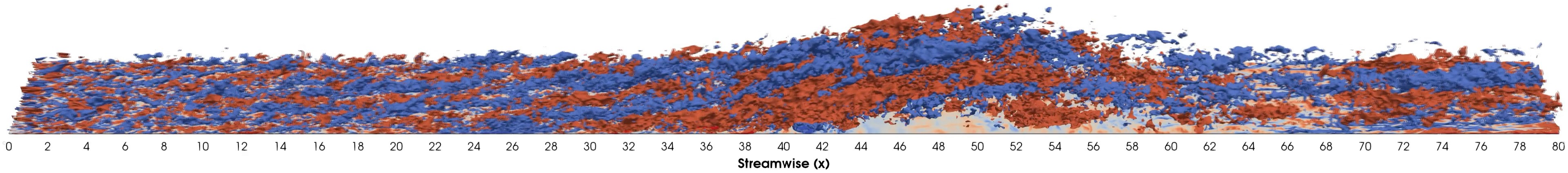
Separated Turbulent Boundary Layer

- **Large Eddy Simulation** of a separated turbulent boundary layer with inlet $Re_\vartheta = 1551$
- LES with relaxation-term filter* (Nek5000)
- Domain size: $80\delta_{99,in}$ x $10\delta_{99,in}$ x $5\delta_{99,in}$

Top BC+: $v_y = V_{top}(x), \ \frac{\partial v_x}{\partial y} = \frac{dV_{top}(x)}{dx}, \ \frac{\partial v_z}{\partial y} = 0$



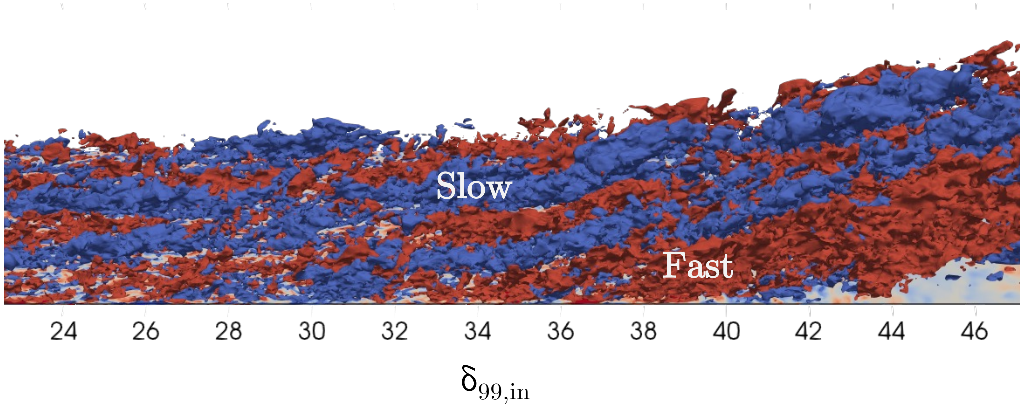
Synthetic Turbulence Generator**, $Re_\vartheta = 1551$

Positive and negative streamwise velocity fluctuation isosurfaces

* Schlatter, Philipp, et al. "LES of transitional flows using the approximate deconvolution model." *International journal of heat and fluid flow* (2004).
** Shur, Michael L., et al. "Synthetic turbulence generators for RANS-LES interfaces in zonal simulations of aerodynamic and aeroacoustic problems." *Flow, turbulence and combustion* (2014).
+ Na, Y., and Parviz Moin. "Direct numerical simulation of a separated turbulent boundary layer." Journal of Fluid Mechanics 374 (1998): 379-405.

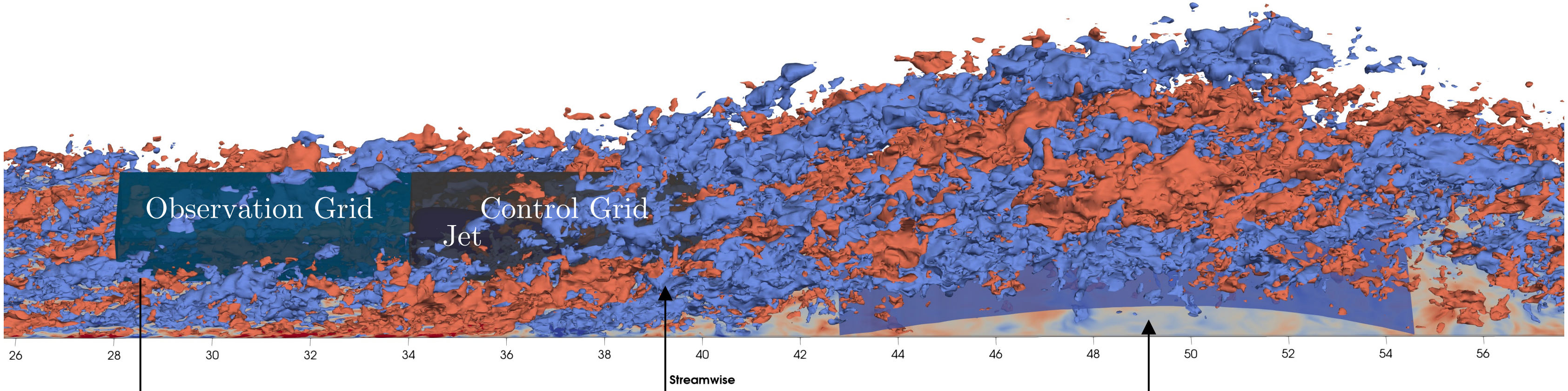
Large-Scale Motions in a Boundary Layer

- Coherent motions in wall-bounded turbulent flows
- Characteristics:
  - Size in the order of the boundary layer thickness
  - Large fraction of the turbulent kinetic energy
  - Significant contribution to average Reynolds shear stresses
- Consist of smaller structures (e.g. hairpin vortices)



Positive and negative streamwise velocity fluctuation isosurfaces at $Re_\theta \cong 2500$

Model Predictive Control



Detect → **Predict** → **Actuate** **Goal**

Detect
Spatial Smoothing

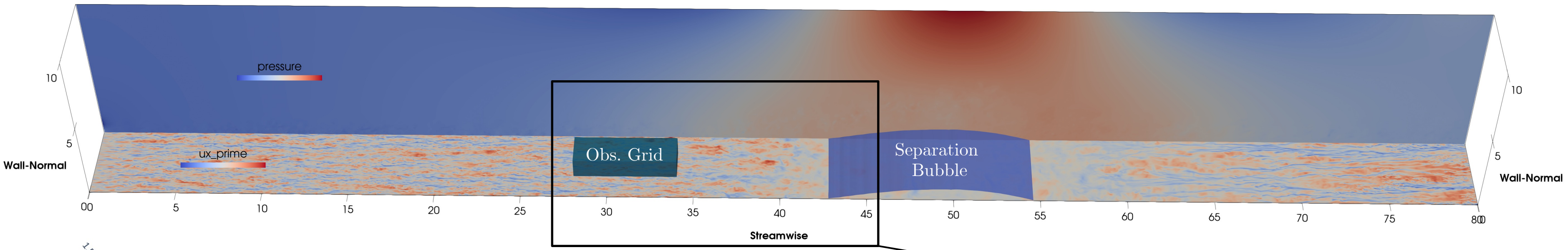
Predict
Taylor's Hypothesis

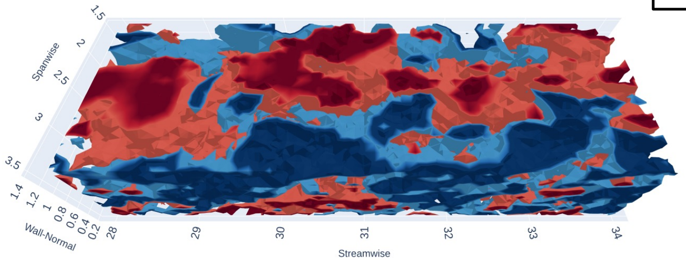
Actuate
Solve Optimal Control

Goal
Minimize Separation Bubble & Control Effort

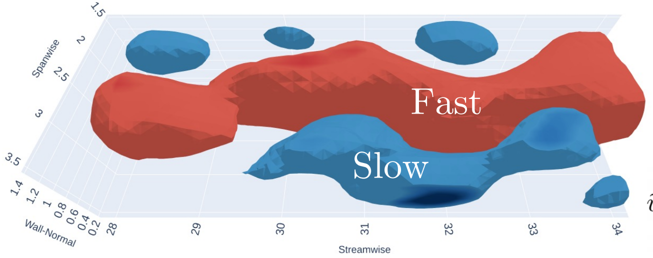
Tsolovikos, Alexandros, et al. "Model Predictive Control of Material Volumes with Application to Vortical Structures." AIAA Journal 59.10 (2021): 4057-4070.

Detecting LSMs



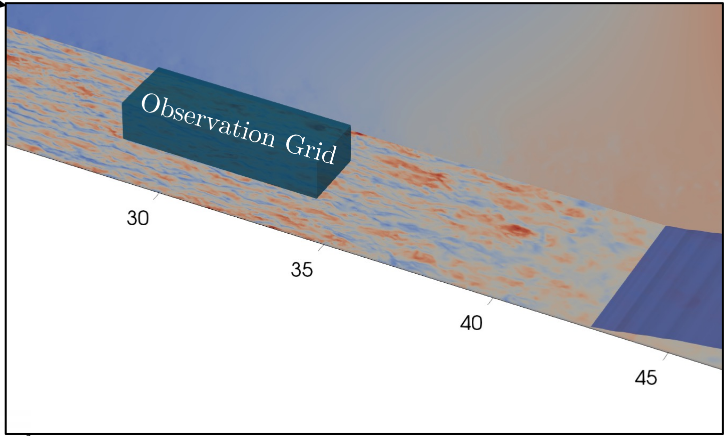


Raw v'_x



Spatially smoothed v'_x

$$\tilde{v}'_x[i,j,k] = \frac{1}{7^3} \sum_{\substack{m_1=-3 \\ m_2=-3 \\ m_3=-3}}^{3} v'_x[i+m_1, j+m_2, k+m_3]$$

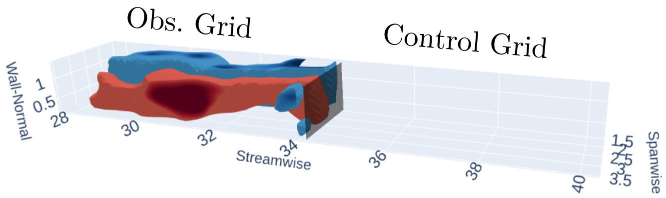


Detect fast and slow zones

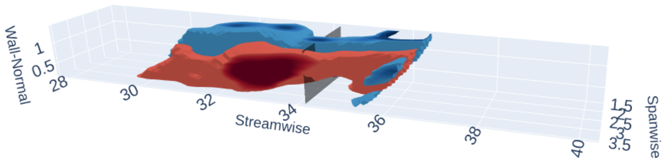
Predicting LSM Movement

Taylor's Hypothesis:
Turbulent eddies are frozen and only advect with the mean velocity field.

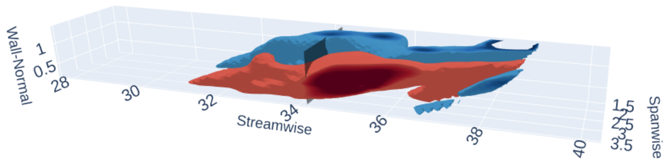
Filtered Observation: $\tilde{v}'_x(0)$



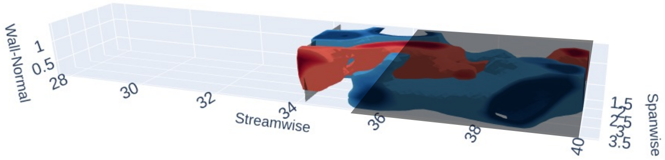
Prediction: $\tilde{v}'_{x,\mathrm{taylor}}(60)$



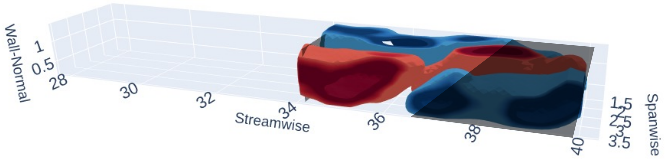
Prediction: $\tilde{v}'_{x,\mathrm{taylor}}(120)$



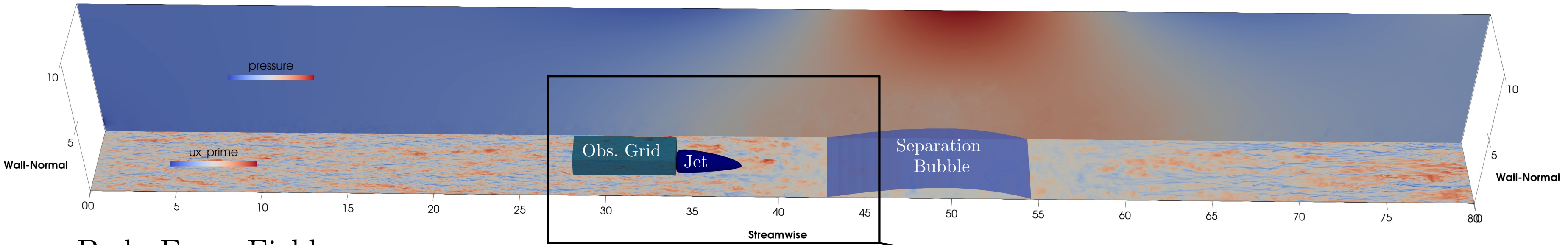
Exact: $\tilde{v}'_x(60)$

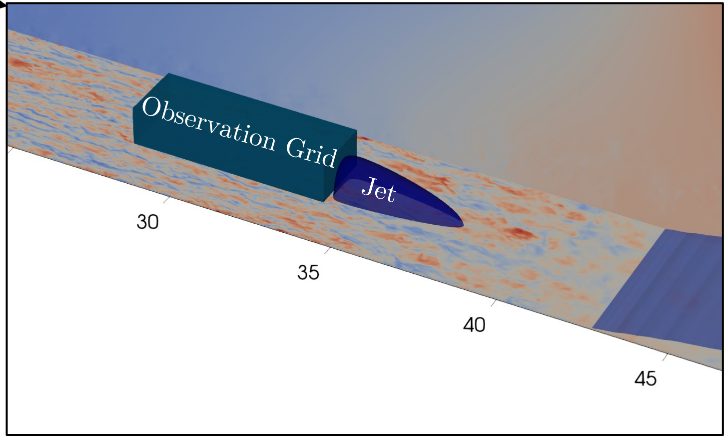


Exact: $\tilde{v}'_x(120)$



Creating Downwash via Body Force





Body Force Field:

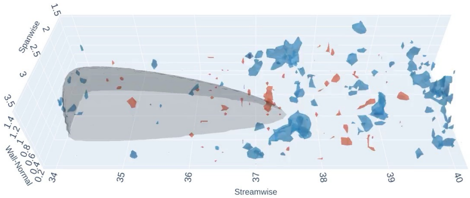
$$f_y(t,x,y,z) \quad = -\tilde{f}(t)g(x,y,z)$$

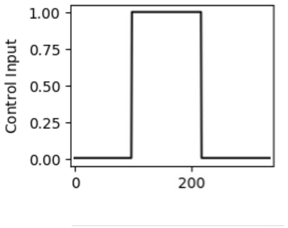
Gamma (x) & Gaussian (y, z) Distribution

Power as Control Input:

$$p(t) = \int_V \mathbf{f}(t,x,y,z) \cdot \mathbf{v}(t,x,y,z)dV$$

$$= -\tilde{f}(t) \int_V g(x,y,z)v_y(t,x,y,z)dV$$



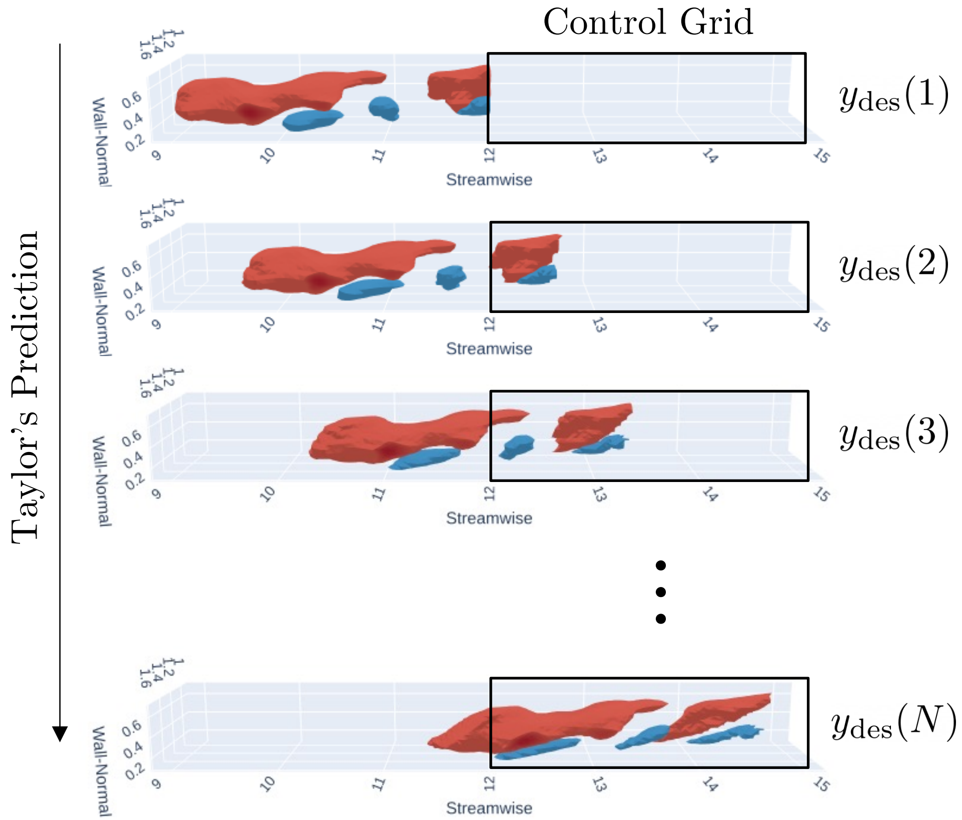


Vertical Body Force to Create Downwash

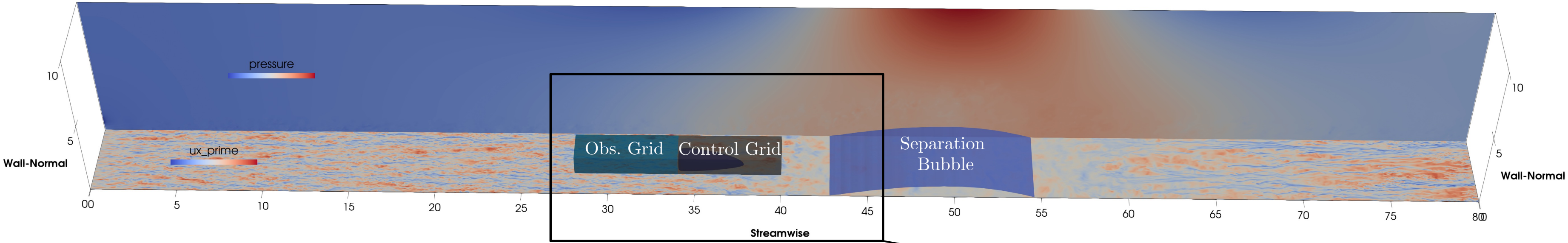
Where to Create Downwash

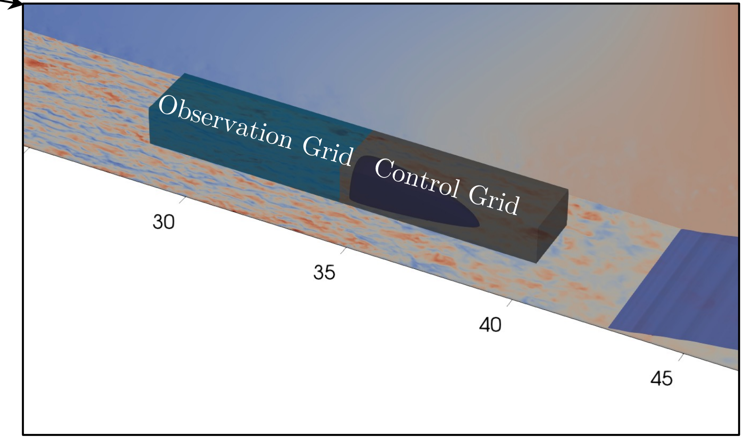
- **Goal**: Create downwash at the predicted location of the fast LSMs while avoiding the slow LSMs

- **Controller**: Determine the optimal input for the next N time steps that satisfies the above goal

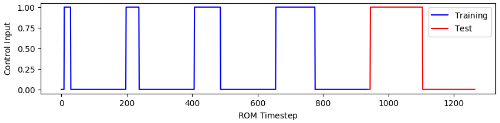


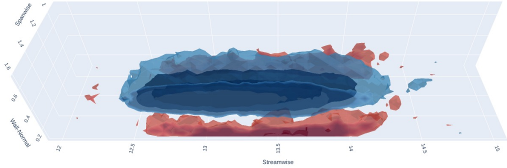
Reduced Order Model of Jet Downwash





- Collect Ensemble-Averaged LES Snapshots





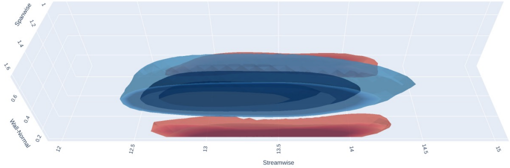
- Compute total least squares Dynamic Mode Decomposition with Control ROM

Control Input

ROM state

$$
\mathbf{x}(t+1) = A\mathbf{x}(t) + Bu(t)
$$
$$
\mathbf{y}(t) = U_{\mathrm{POD}}\mathbf{x}(t) + \mathbf{y}_{\mathrm{mean}}
$$

Wall-normal vel.



* Dawson, Scott, et al. "Characterizing and correcting for the effect of sensor noise in the dynamic mode decomposition." *Experiments in Fluids* 57.3 (2016): 1-19.

Where to Create Downwash

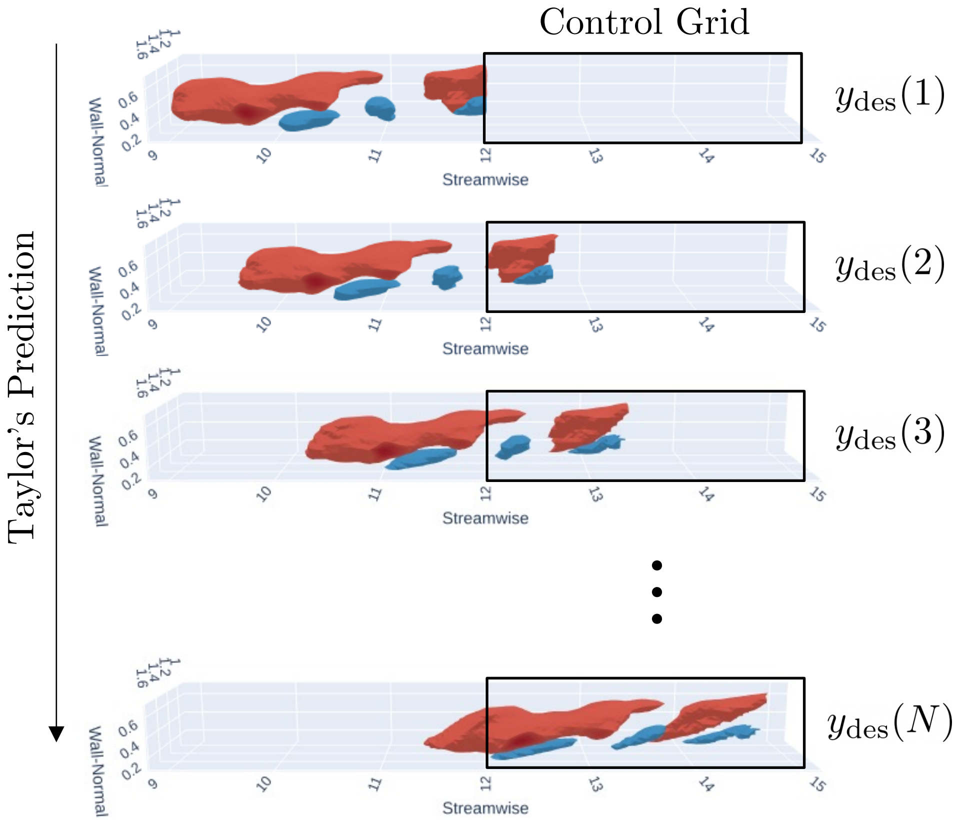
- Desired Output: $\boxed{y_{\text{des}}(t) = \lambda \tilde{v}'_{x,\text{taylor}}(t)}$

- Optimal Controller:

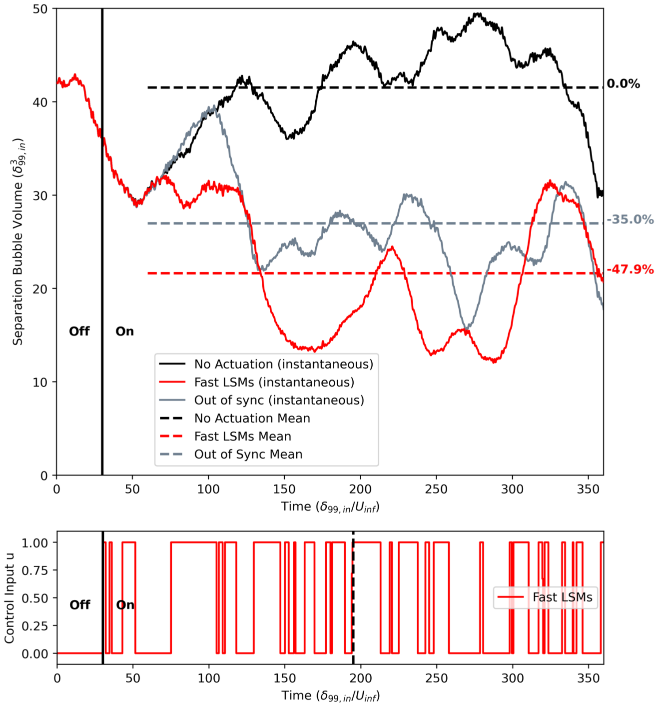
$$
\begin{aligned}
U^* = \arg\min_{U} \quad & \sum_{t=0}^{N-1} \|u(t)\|_R^2 + \|y(t+1) - y_{\text{des}}(t+1)\|_Q^2 \\
\text{subject to} \quad & x(t+1) = Ax(t) + Bu(t) \\
& y(t) = Cx(t) + \bar{y} \\
& x(0) = x_0 \\
& 0 \leq u(t) \leq 1 \\
& u(0) = u(1) = \ldots = u(T-1) \\
& u(T) = u(T+1) = \ldots = u(2T-1) \\
& \vdots \\
& u(N-T) = u(N-T-1) = \ldots = u(N-1)
\end{aligned}
$$

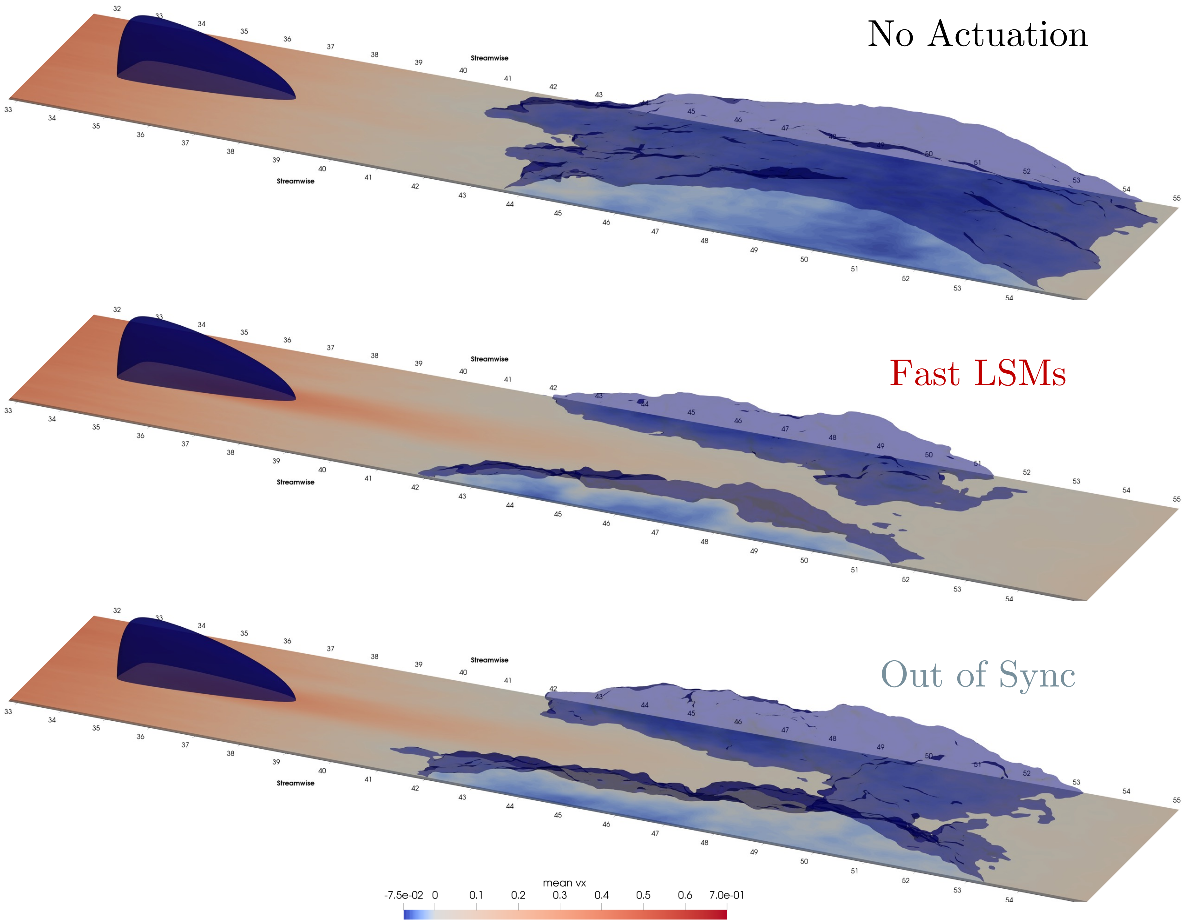
Reduced Order Model

Quadratic Program with Inequality Constraints

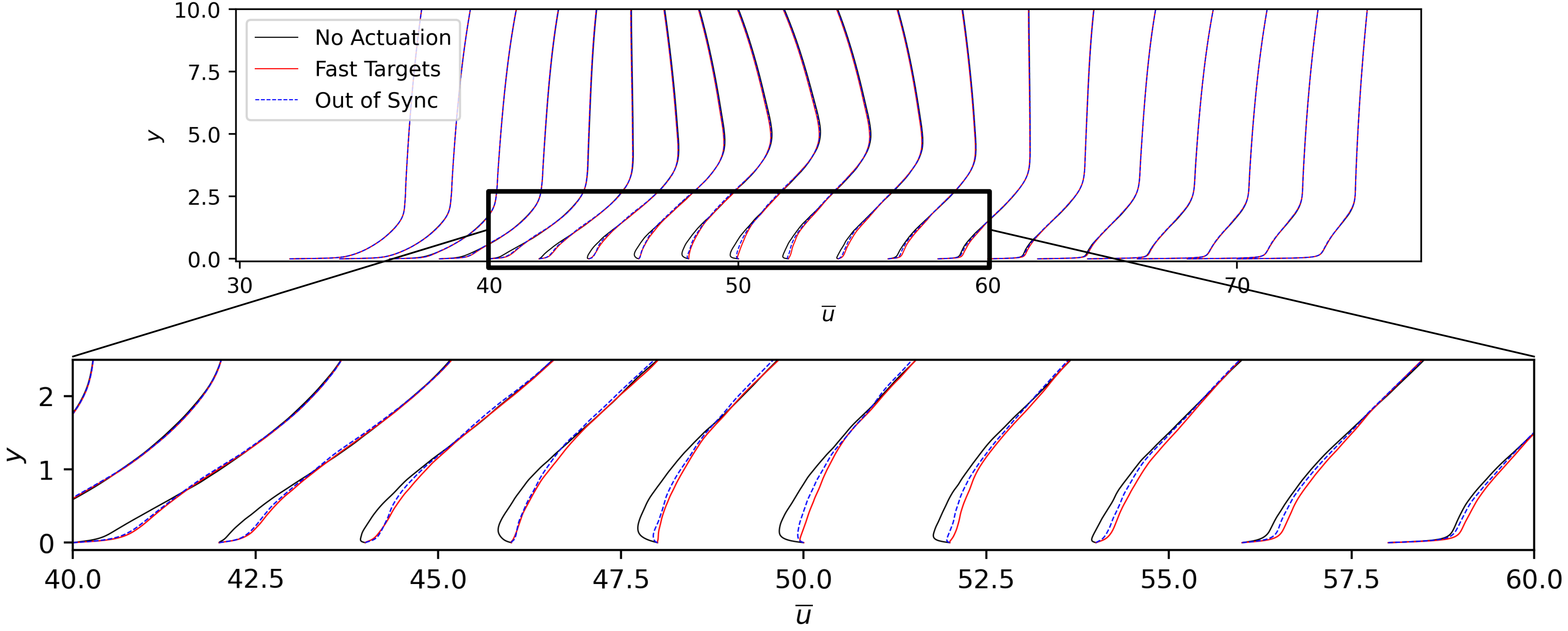


Fast LSMs

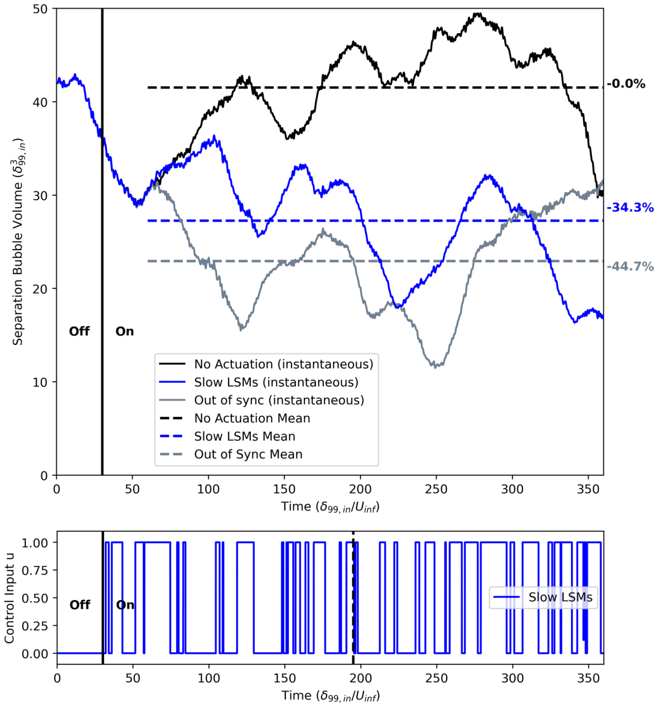

Separation Bubble Volume ($\delta^3_{99,in}$)
Time ($\delta_{99,in}/U_{inf}$)
Off
On
0.0%
-35.0%
-47.9%
No Actuation (instantaneous)
Fast LSMs (instantaneous)
Out of sync (instantaneous)
No Actuation Mean
Fast LSMs Mean
Out of Sync Mean
Control Input u
Off
On
Fast LSMs
Time ($\delta_{99,in}/U_{inf}$)


No Actuation
Fast LSMs
Out of Sync
Streamwise
mean vx
-7.5e-02
0
0.1
0.2
0.3
0.4
0.5
0.6
7.0e-01

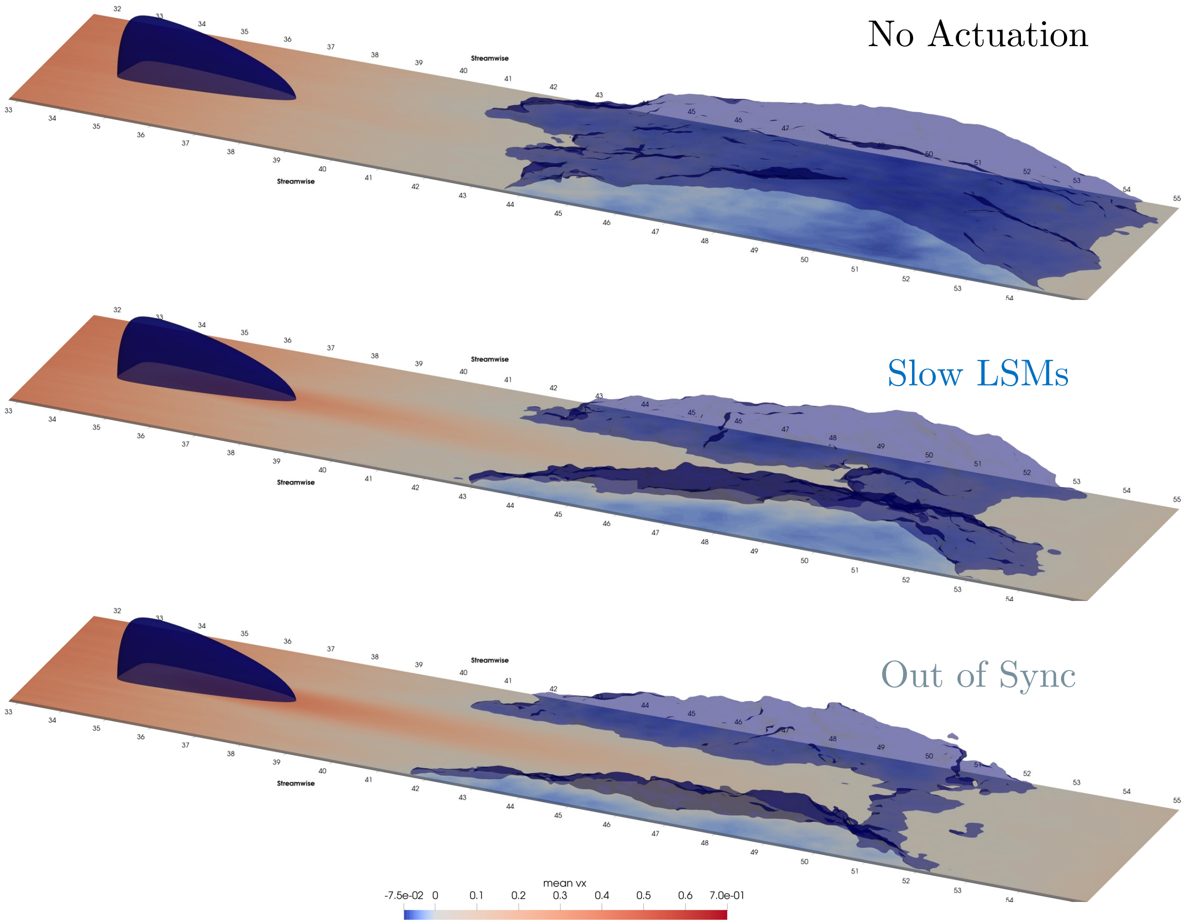
Targeting Fast LSMs



Slow LSMs



Separation Bubble Volume ($\delta_{99,in}^3$)
Time ($\delta_{99,in}/U_{inf}$)
Off
On
-0.0%
-34.3%
-44.7%
No Actuation (instantaneous)
Slow LSMs (instantaneous)
Out of sync (instantaneous)
No Actuation Mean
Slow LSMs Mean
Out of Sync Mean
Control Input u
Slow LSMs

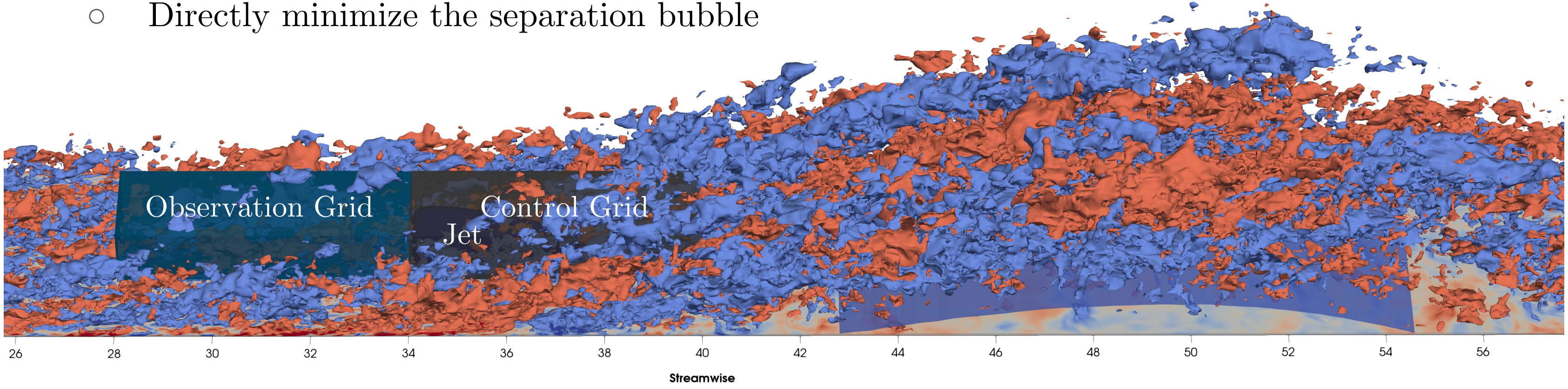


No Actuation
Slow LSMs
Out of Sync
Streamwise
mean vx
-7.5e-02 0 0.1 0.2 0.3 0.4 0.5 0.6 7.0e-01

Future Work

- Explore parametric space:
  - Size & location of jet/observation grid/control grid
  - Optimal control parameters (Q, R, N)
- Reinforcement learning (model-free) control
  - Directly minimize the separation bubble

alextsolovikos.github.io



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